Modele Markov Ascunse

Laura Florea

Date

La un moment de timp, *t*, am:

- o observatie, Y_t
- o stare ascunsa (pe care nu o vad, ci o pot infera), X_t

Am date:

- un set de astfel de observatii, $Y_{1:t}$
- probabilitatile initiale ale starilor ascunse, $P(X_0)$
- probabilitatile de emisie a observatiei curente din fiecare stare ascunsa, $P(Y_t|X_t)$, pentru orice t
- probabilitatile de trecere dintr-o stare ascunsa in alta stare ascunsa, $P(X_t|X_{t-1})$

Scop

- Avand un set de observatii, care este probabilitatea ca starea ascunsa in care ma aflu sa fie X_t , $P(X_t | Y_{1:t})$

Algoritm: predictie

$$P(X_t \mid Y_{1:t-1}) = \sum_{X_{t-1}} P(X_t, X_{t-1} \mid Y_{1:t-1}) =$$

$$= \sum_{x_{t-1}} P(X_t \mid X_{t-1}, Y_{1:t-1}) P(X_{t-1} \mid Y_{1:t-1})$$

Am folosit:

$$P(A \mid C) = \sum_{B} P(A, B \mid C)$$

$$P(A, B \mid C) = P(A \mid B, C) P(B \mid C)$$

Algoritm: predictie

$$P(X_t \mid Y_{1:t-1}) = \sum_{X_{t-1}} P(X_t, X_{t-1} \mid Y_{1:t-1}) =$$

$$= \sum_{X_{t-1}} P(X_t \mid X_{t-1}, Y_{1:t-1}) P(X_{t-1} \mid Y_{1:t-1})$$

Starea ascunsa actuala:

- depinde **doar** de starea ascunsa trecuta si de observatia actuala.
- NU depinde de observatiile trecute.

$$P(X_t \mid Y_{1:t-1}) == \sum_{X_{t-i}} P(X_t \mid X_{t-1}) P(X_{t-1} \mid Y_{1:t-1})$$

Algoritm: update Bayesian

$$P(X_t \mid Y_{1:t}) = P(X_t \mid Y_t, Y_{1:t-1}) =$$

$$= \frac{P(Y_t \mid X_t, Y_{1:t-1}) P(X_t \mid Y_{1:t-1})}{\sum_{X_t} P(Y_t \mid X_t, Y_{1:t-1}) P(X_t \mid Y_{1:t-1})}$$

Am folosit:

$$P(A \mid B, C) = \frac{P(B \mid A, C) P(A \mid C)}{P(B \mid C)}$$

Algoritm: update Bayesian

$$P(X_t \mid Y_{1:t}) = \frac{P(Y_t \mid X_t, Y_{1:t-1}) P(X_t \mid Y_{1:t-1})}{\sum_{X_t} P(Y_t \mid X_t, Y_{1:t-1}) P(X_t \mid Y_{1:t-1})}$$

Dar observatia curenta depinde doar de starea curenta.
NU depinde de observatiile anterioare.

$$P(X_t \mid Y_{1:t}) = \frac{P(Y_t \mid X_t) P(X_t \mid Y_{1:t-1})}{\sum_{X_t} P(Y_t \mid X_t) P(X_t \mid Y_{1:t-1})}$$

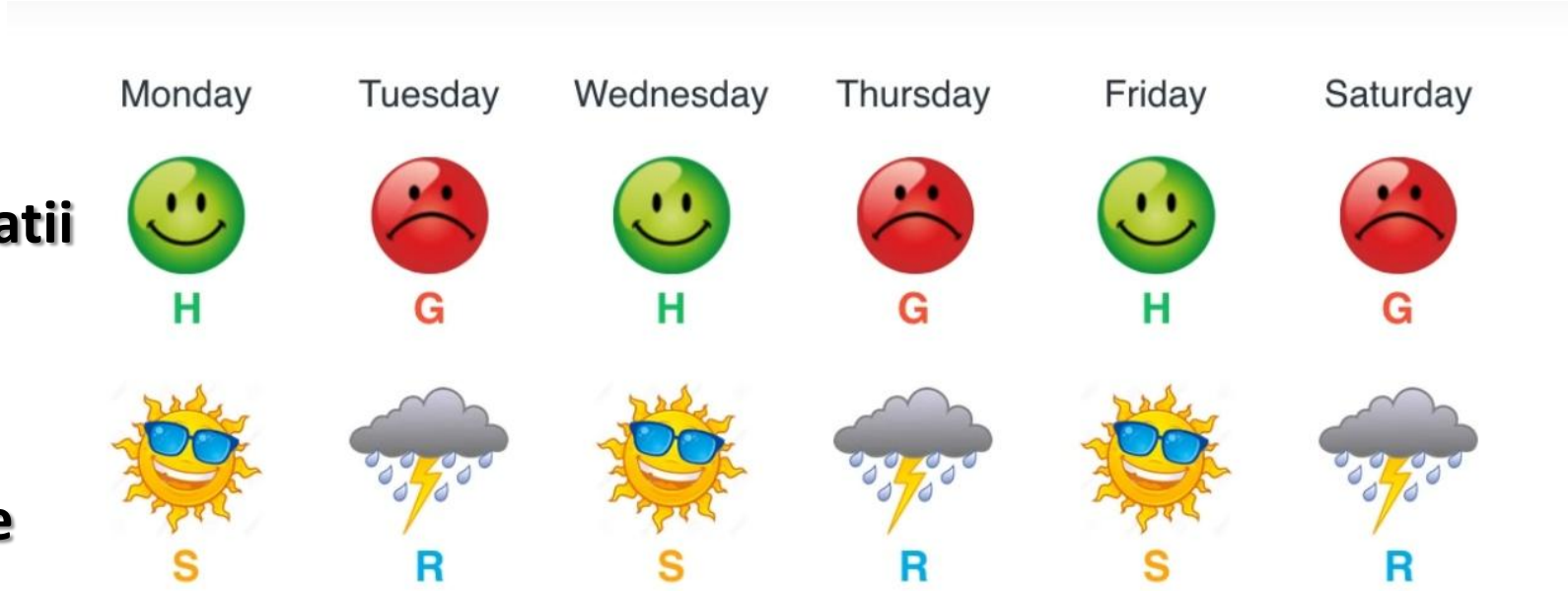
Algoritm: update Bayesian

Cunoscuta

Calculata la pasul anterior

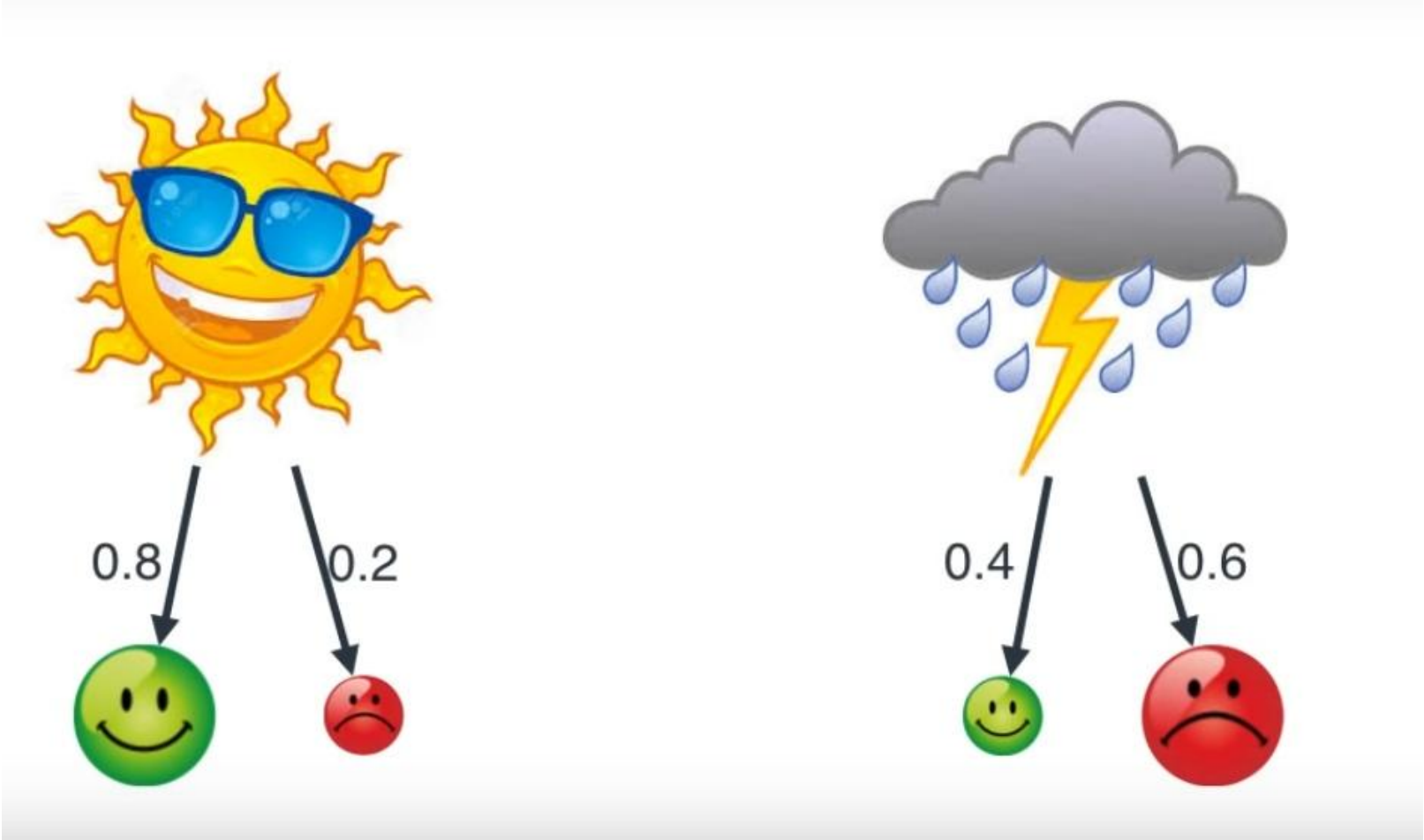
$$P(X_t \mid Y_{1:t}) = \frac{P(Y_t \mid X_t) P(X_t \mid Y_{1:t-1})}{\sum_{X_t} P(Y_t \mid X_t) P(X_t \mid Y_{1:t-1})}$$

Exemplu

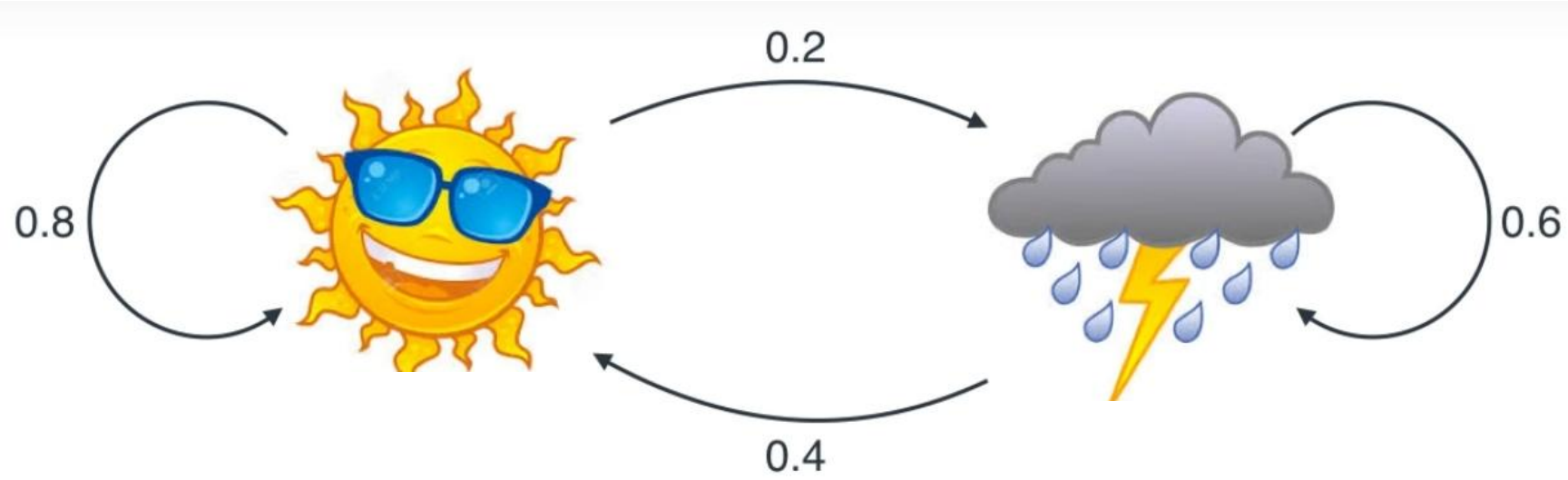


Exemplu preluat de la Luis Serrano, **A friendly introduction to Bayes Theorem and Hidden Markov Models**
https://www.youtube.com/watch?v=kqSzLo9fenk

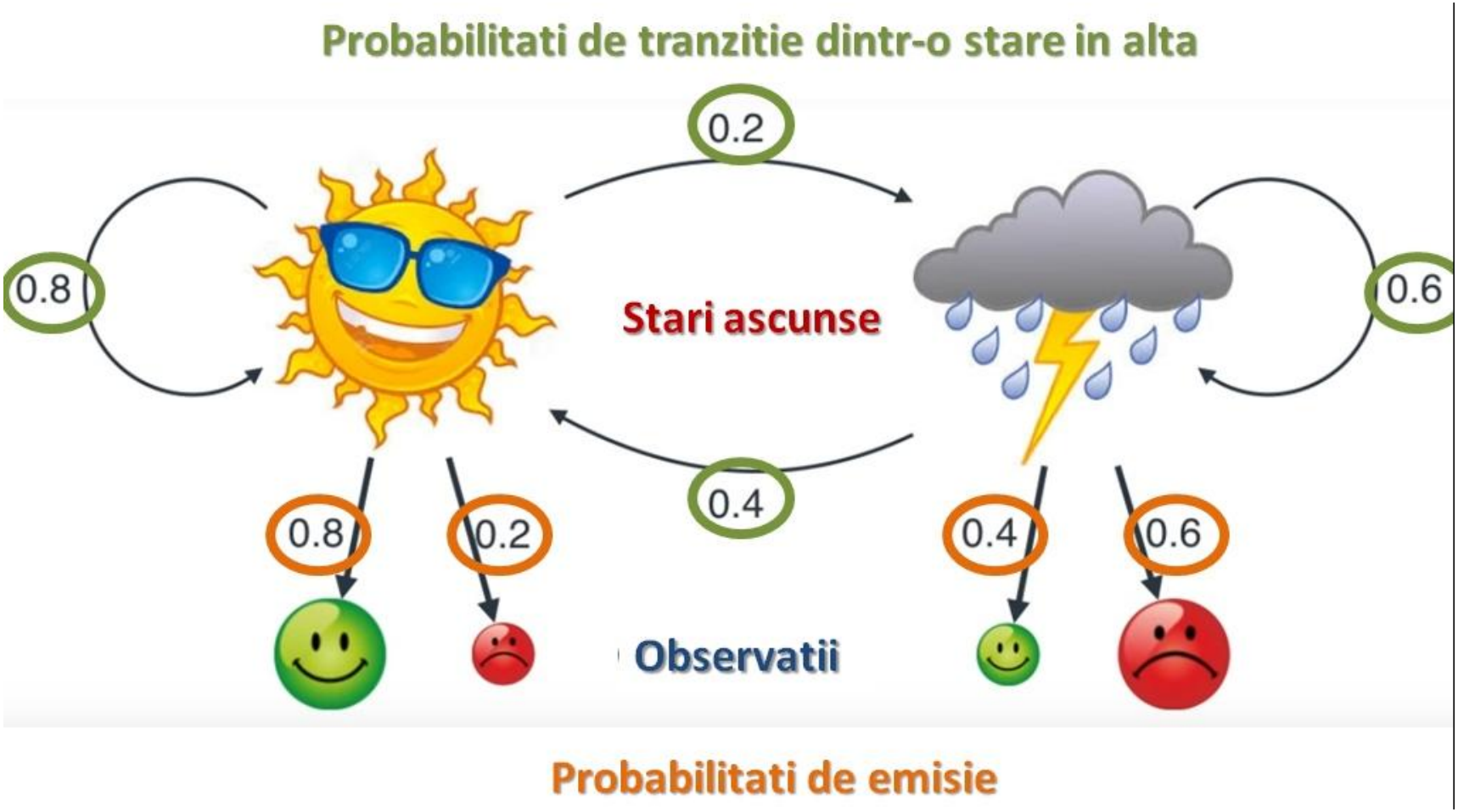
Probabilitati de emisie



Probabilitati de tranzitie



Date


Probabilitati de tranzitie dintr-o stare in alta
0.2
0.8
0.6
Stari ascunse
0.4
0.8
0.2
0.4
0.6
Observatii
Probabilitati de emisie

Cum aflu probabilitatile?

Prin masuratori anterioare (antrenare)





8 0.8



2 0.2



2 0.4



3 0.6

Cum aflu probabilitatile?





8 0.8



2 0.2

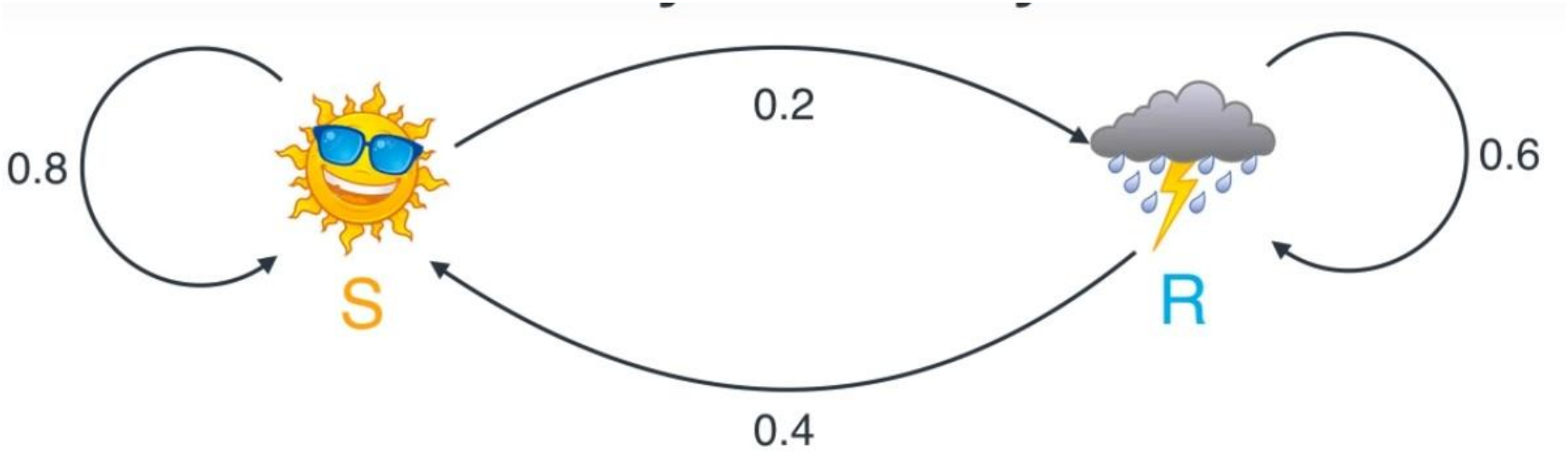


2 0.4



3 0.6

Cum aflu probabilitatile?



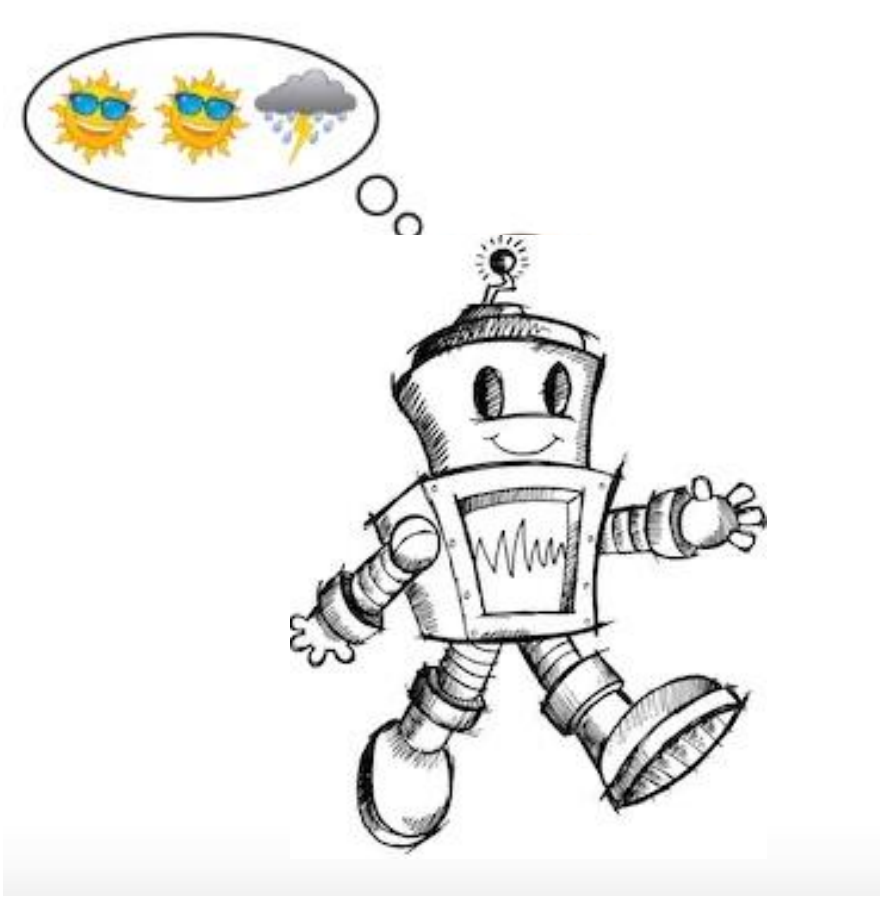
$S = 0.8S + 0.4R$

$S + R = 1$

$R = 0.2S + 0.6R$

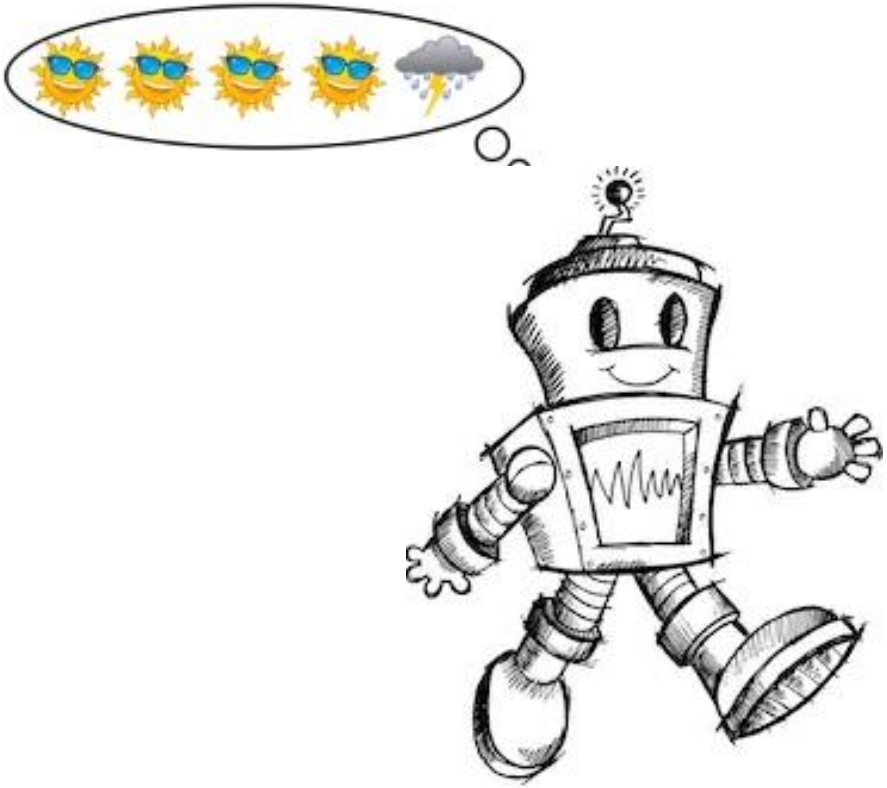
$S = 2/3$
$R = 1/3$

Daca nu stie decat probabilitatile initiale…



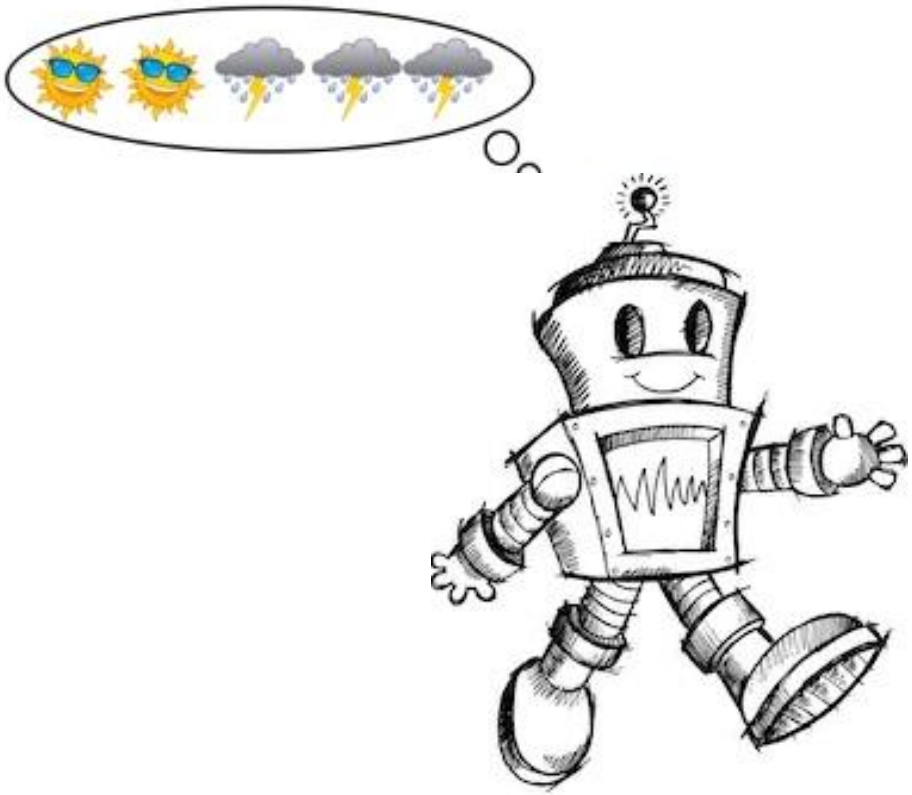


Daca stie si observatia actuala



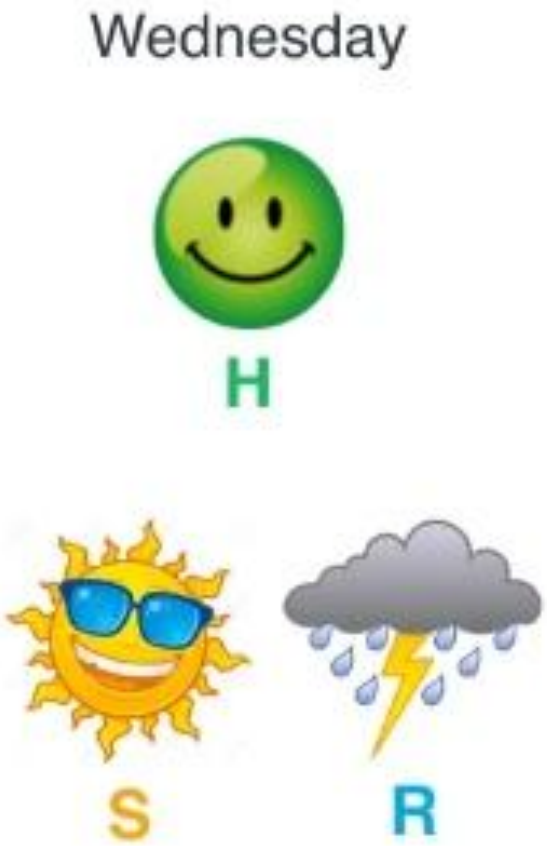


Daca stie si observatia actuala

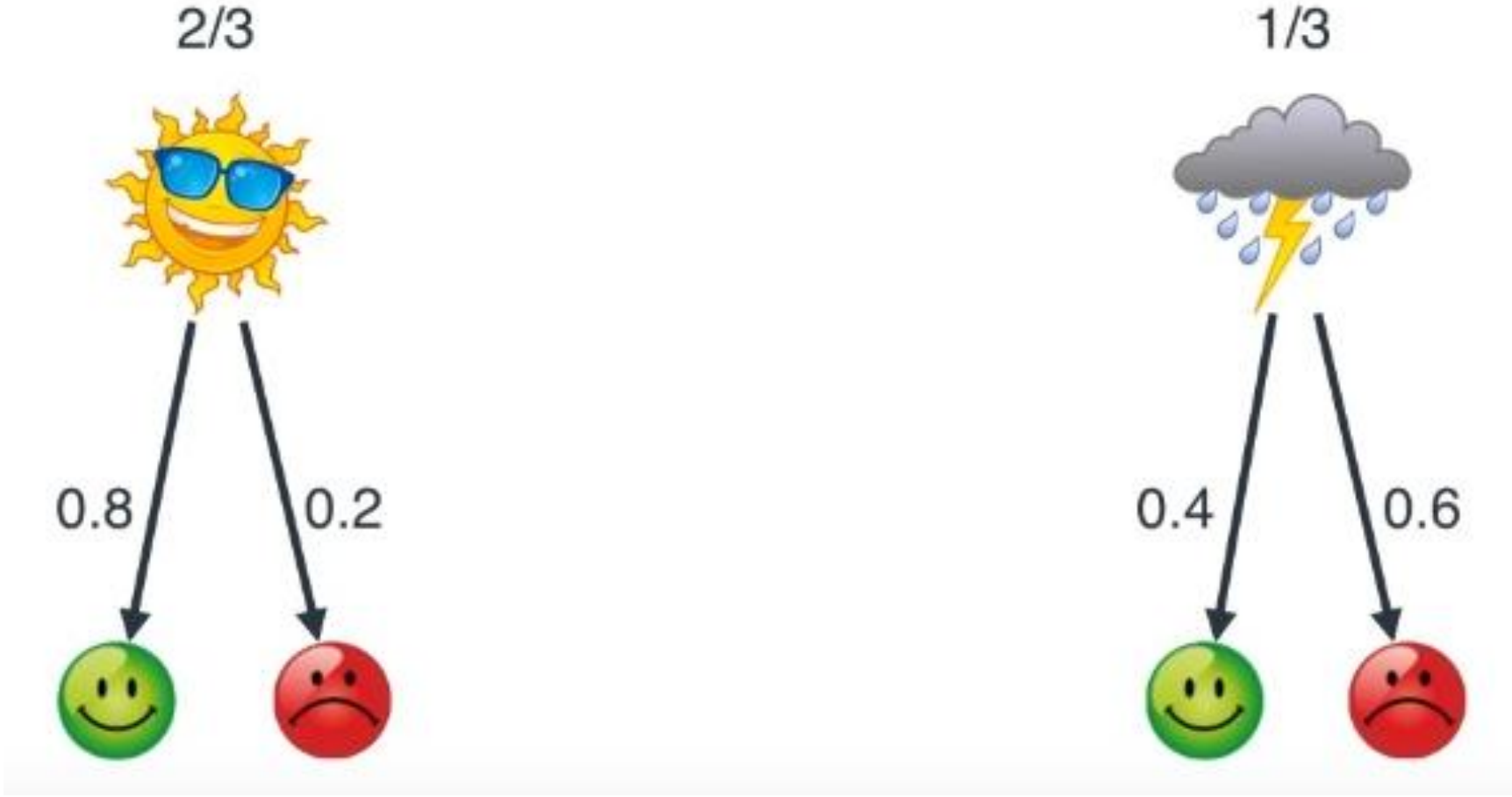




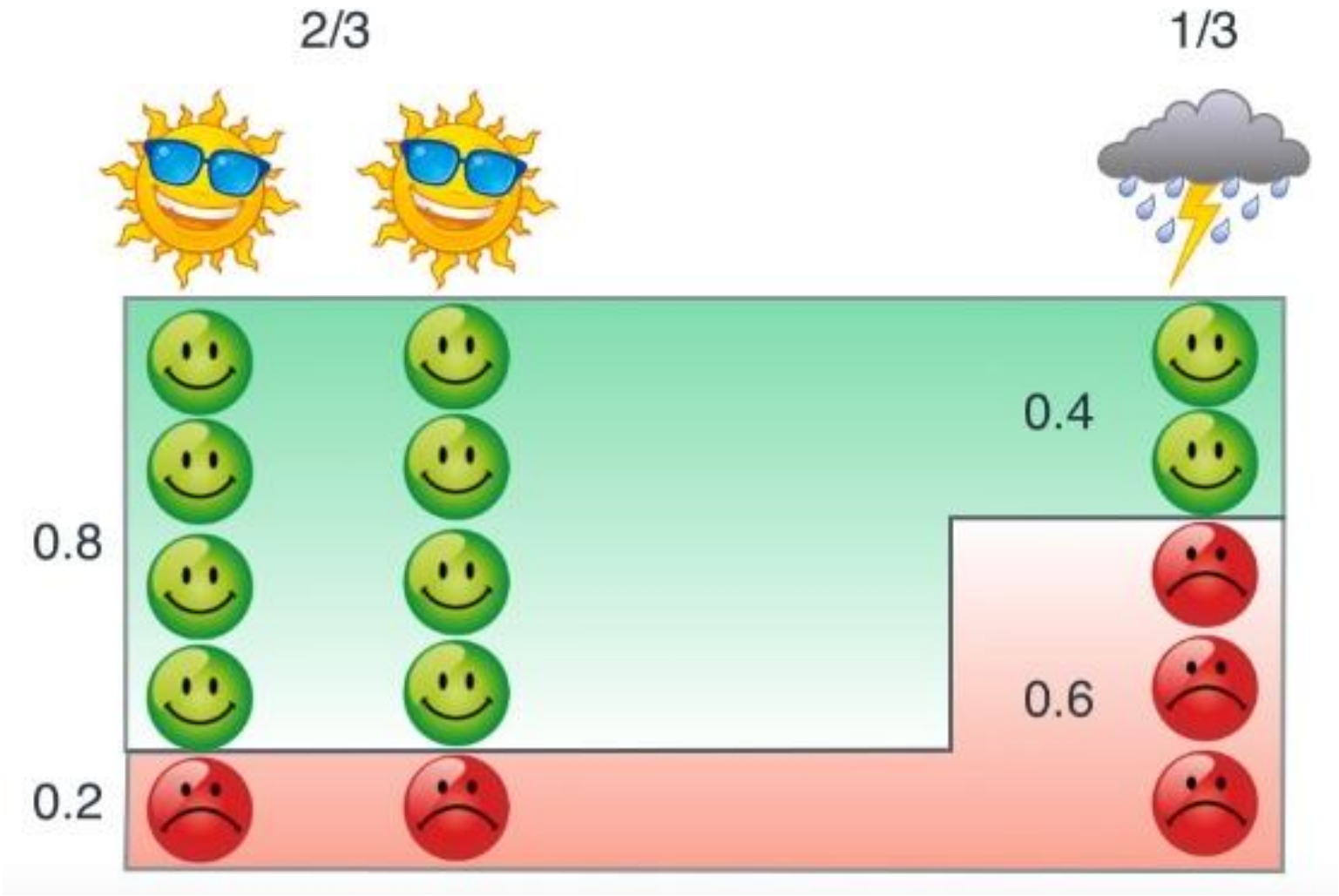
O singura observatie: 2 cazuri posibile

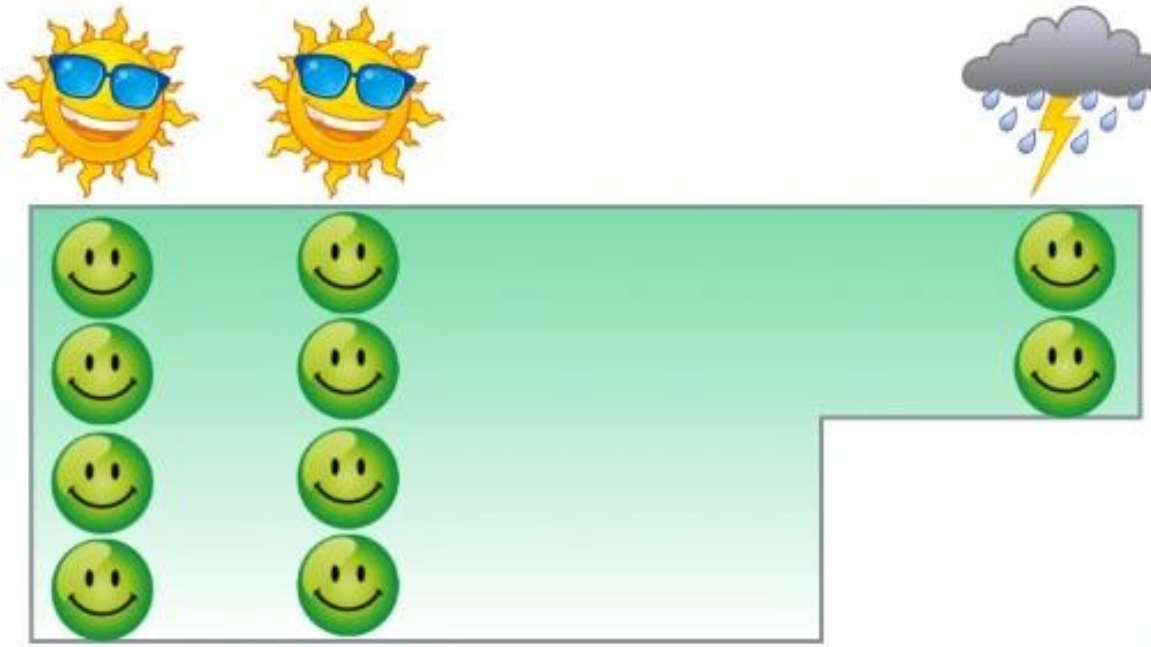


Se stiu:




2/3
1/3
0.8
0.2
0.4
0.6


2/3
1/3
0.4
0.8
0.6
0.2



If 🙂

$$P(☀ \mid 🙂) = \frac{8}{10}$$

$$P(⛈ \mid 🙂) = \frac{2}{10}$$





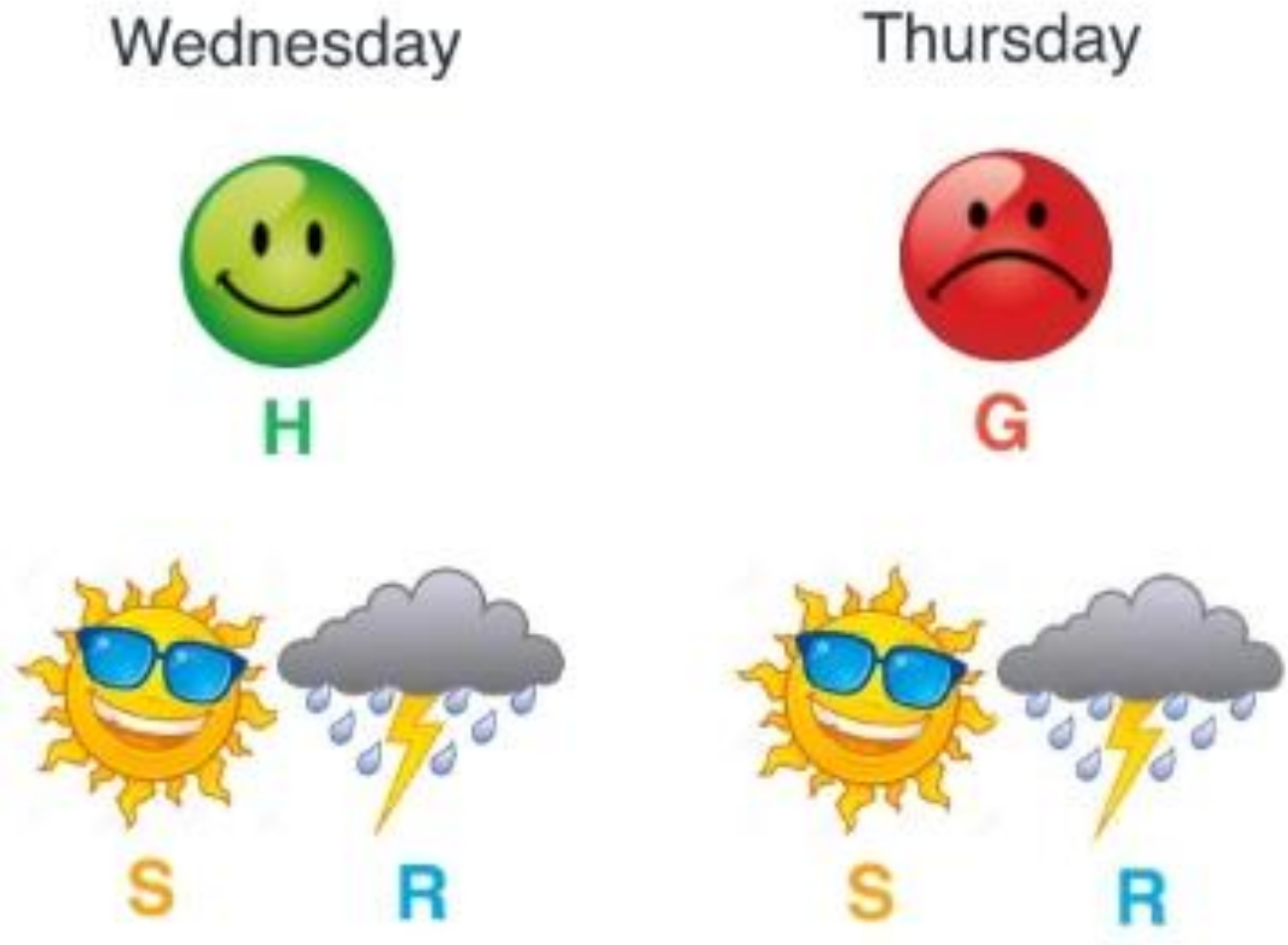
If 



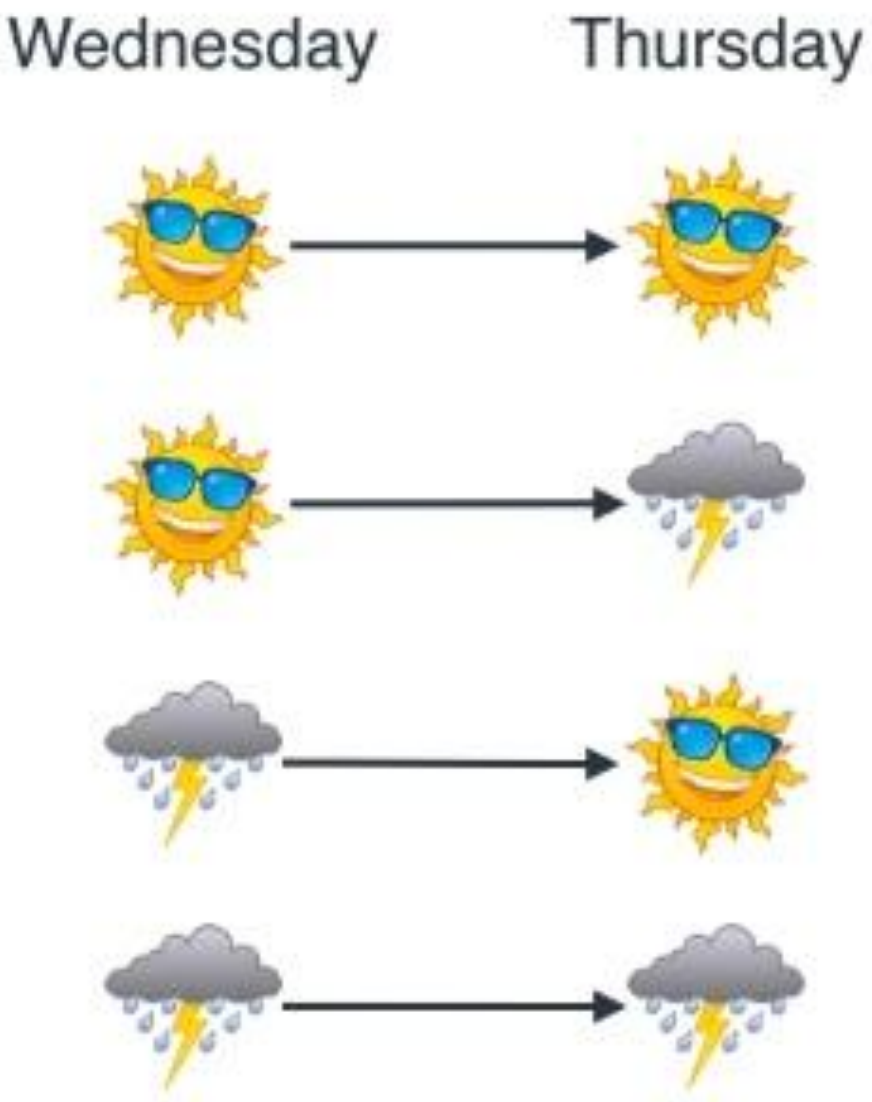
$$P(\text{sun} \mid \text{sad}) = \frac{2}{5}$$

$$P(\text{storm} \mid \text{sad}) = \frac{3}{5}$$

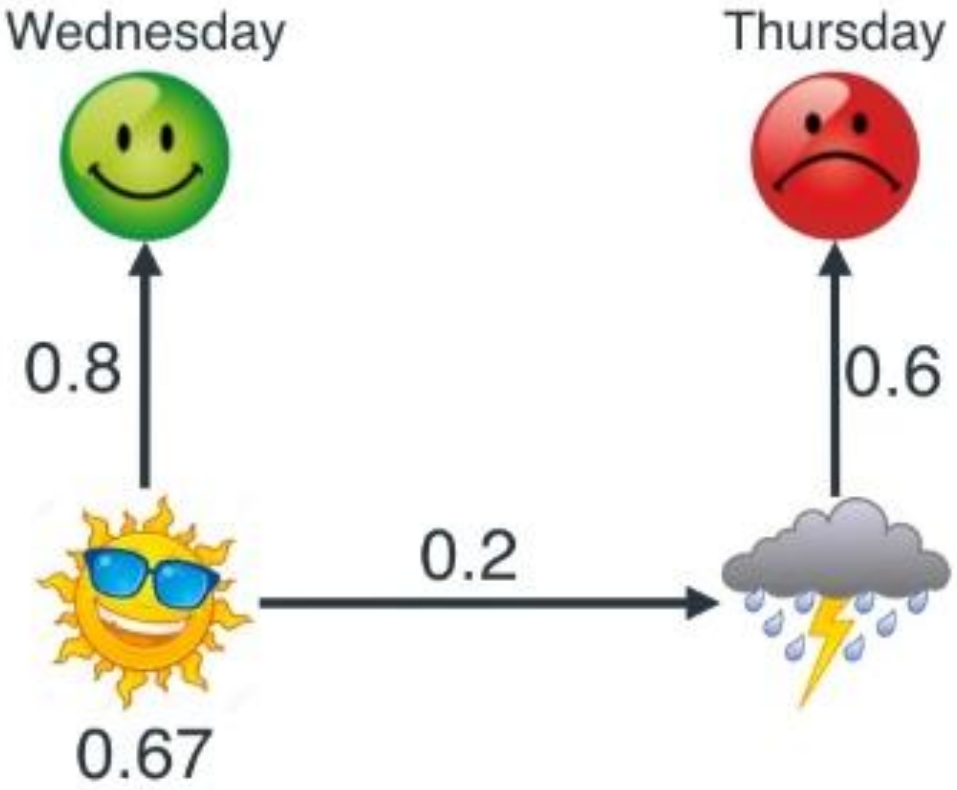
Am doua observatii



Exista 4 cazuri

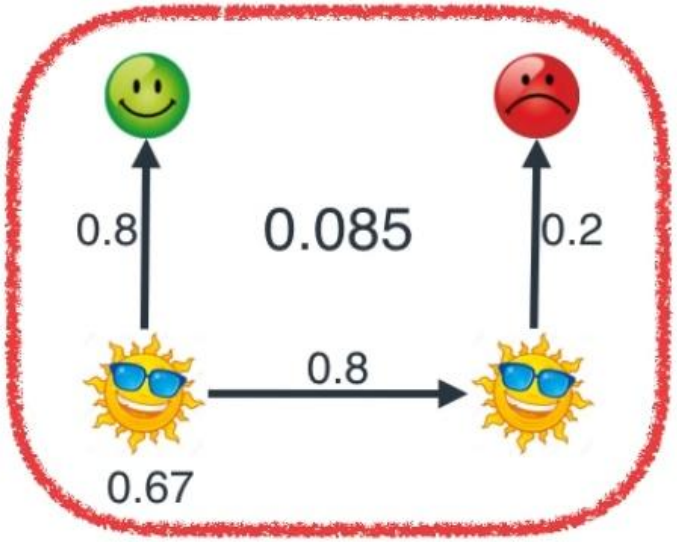


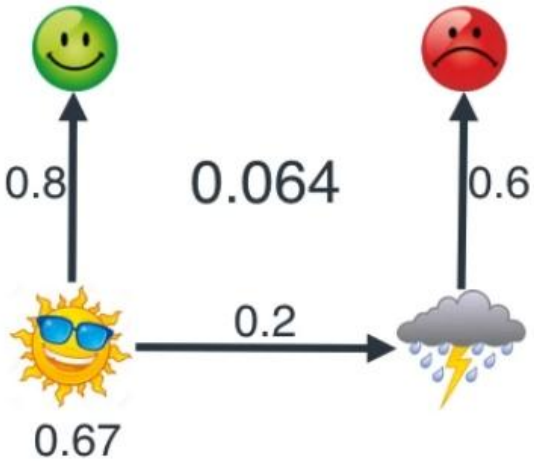
Calculez pentru fiecare probabilitatea:

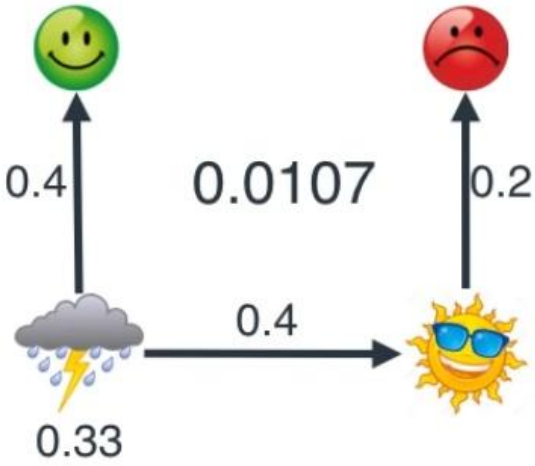


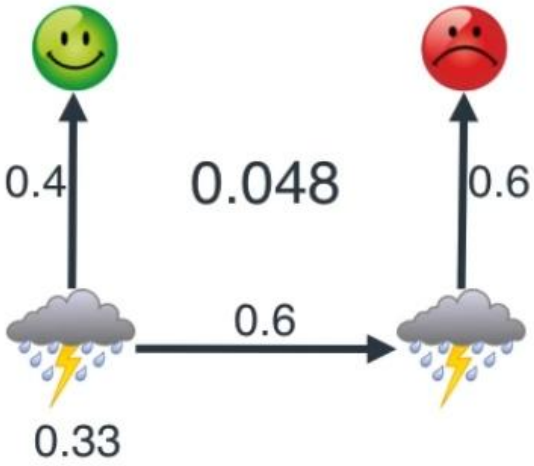
0.064

Si aleg varianta de maxima verosimilitate





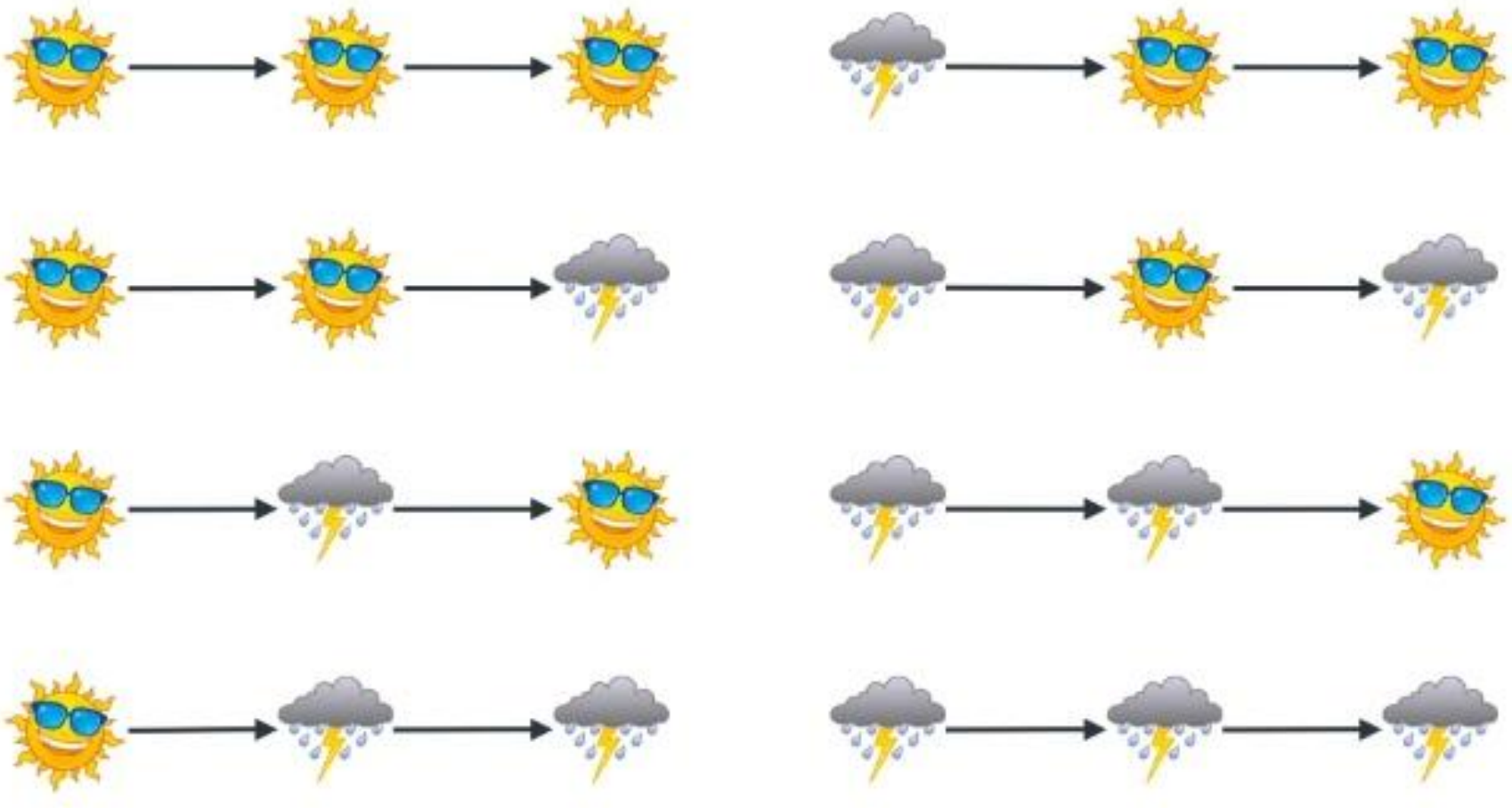




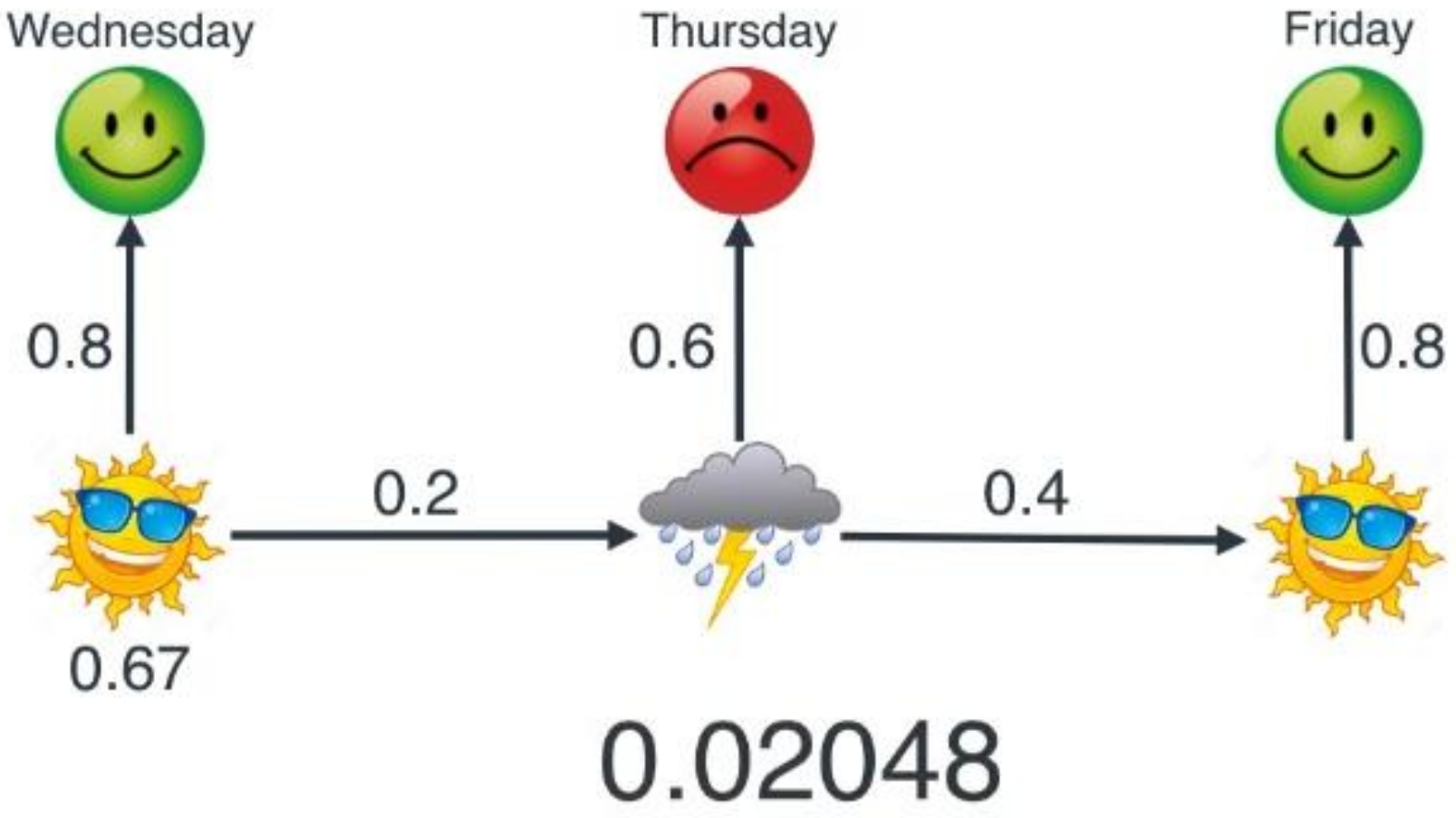
Am 3 observatii:



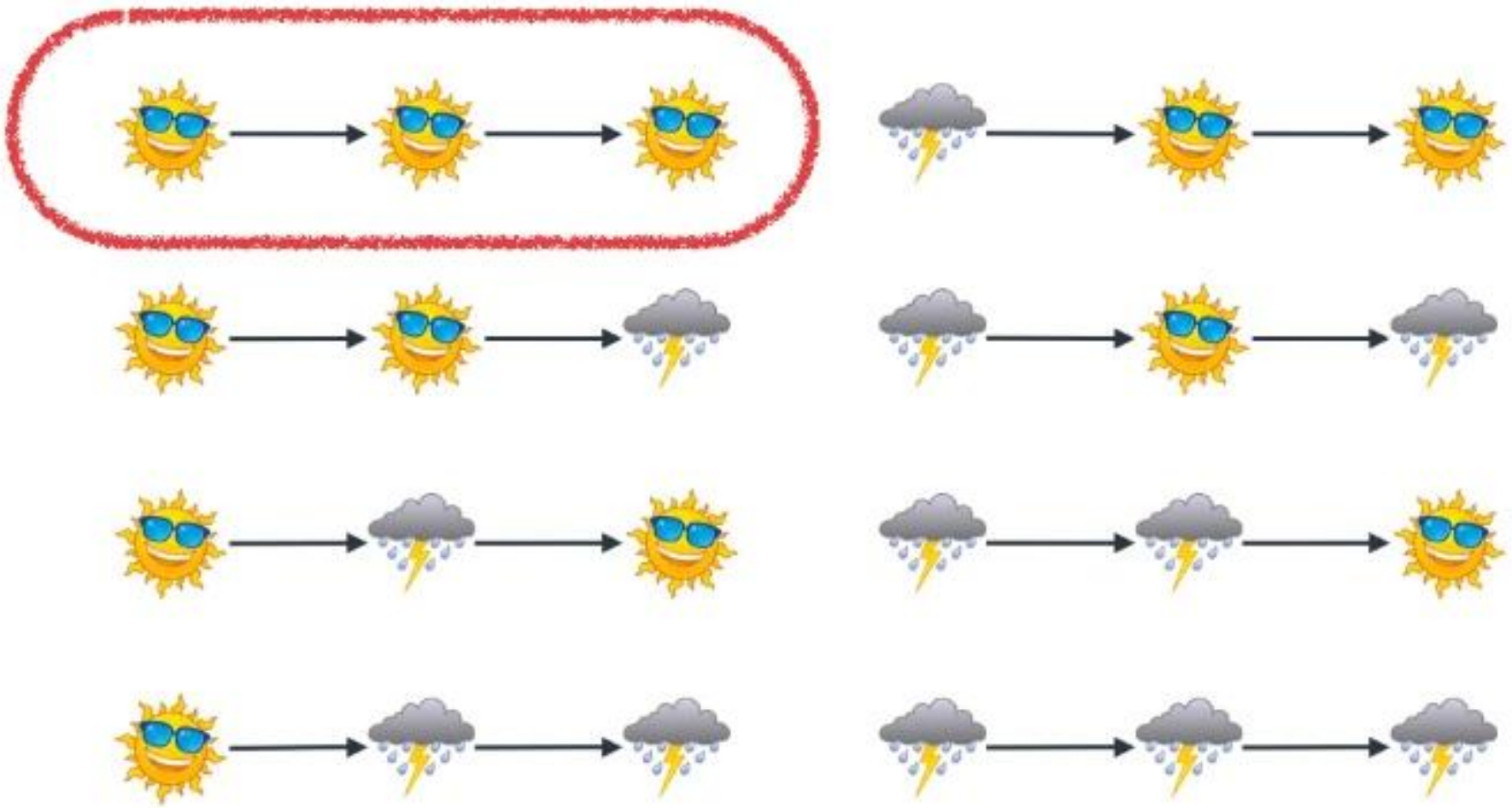
Am 2^3=8 posibilitati



Calculez pentru fiecare probabilitatea:

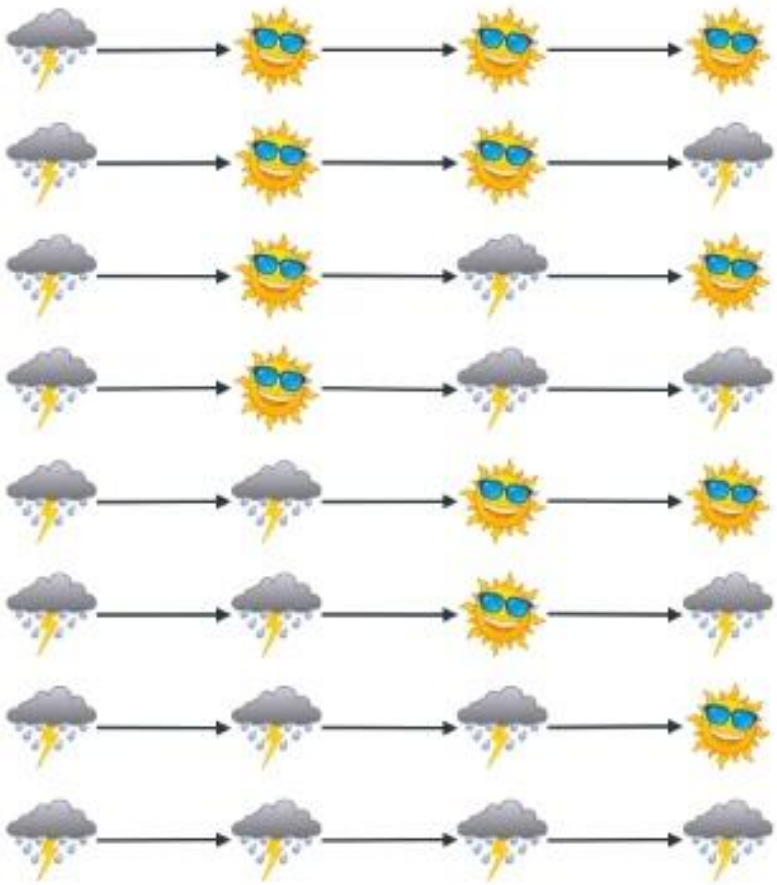


Si aleg varianta de maxima verosimilitate

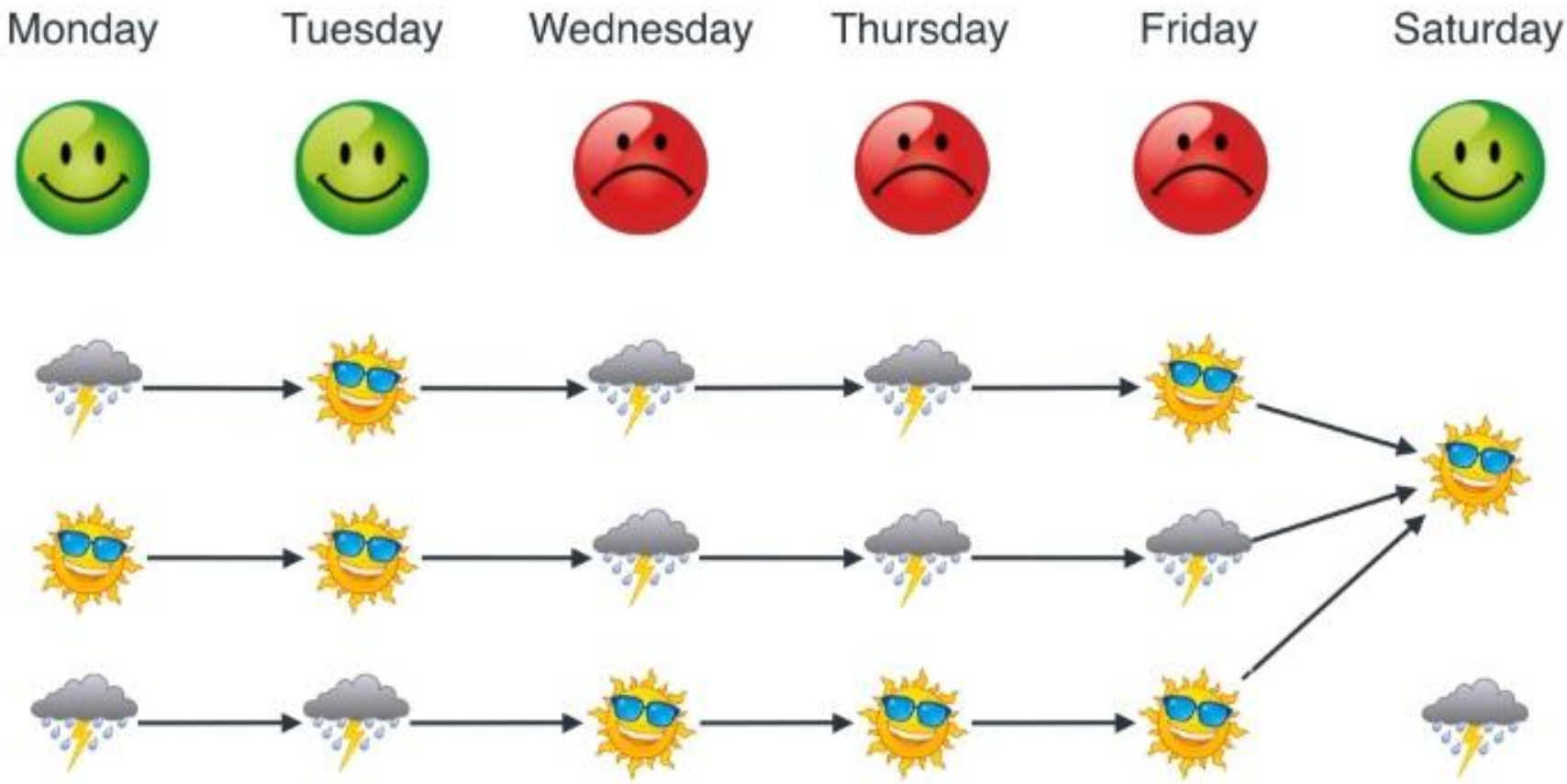


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Algoritmul Viterbi



Algoritmul Viterbi



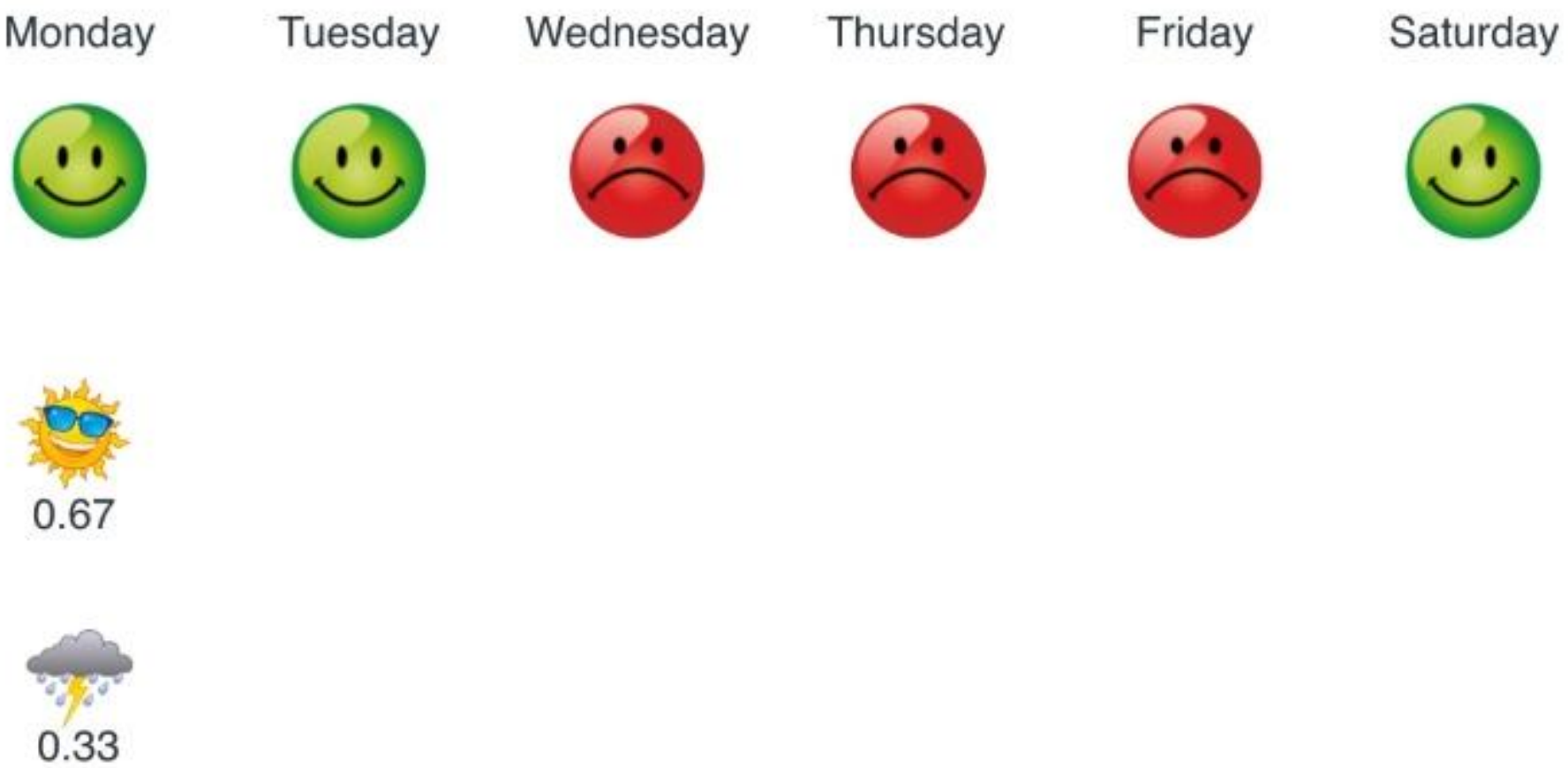
Algoritmul Viterbi



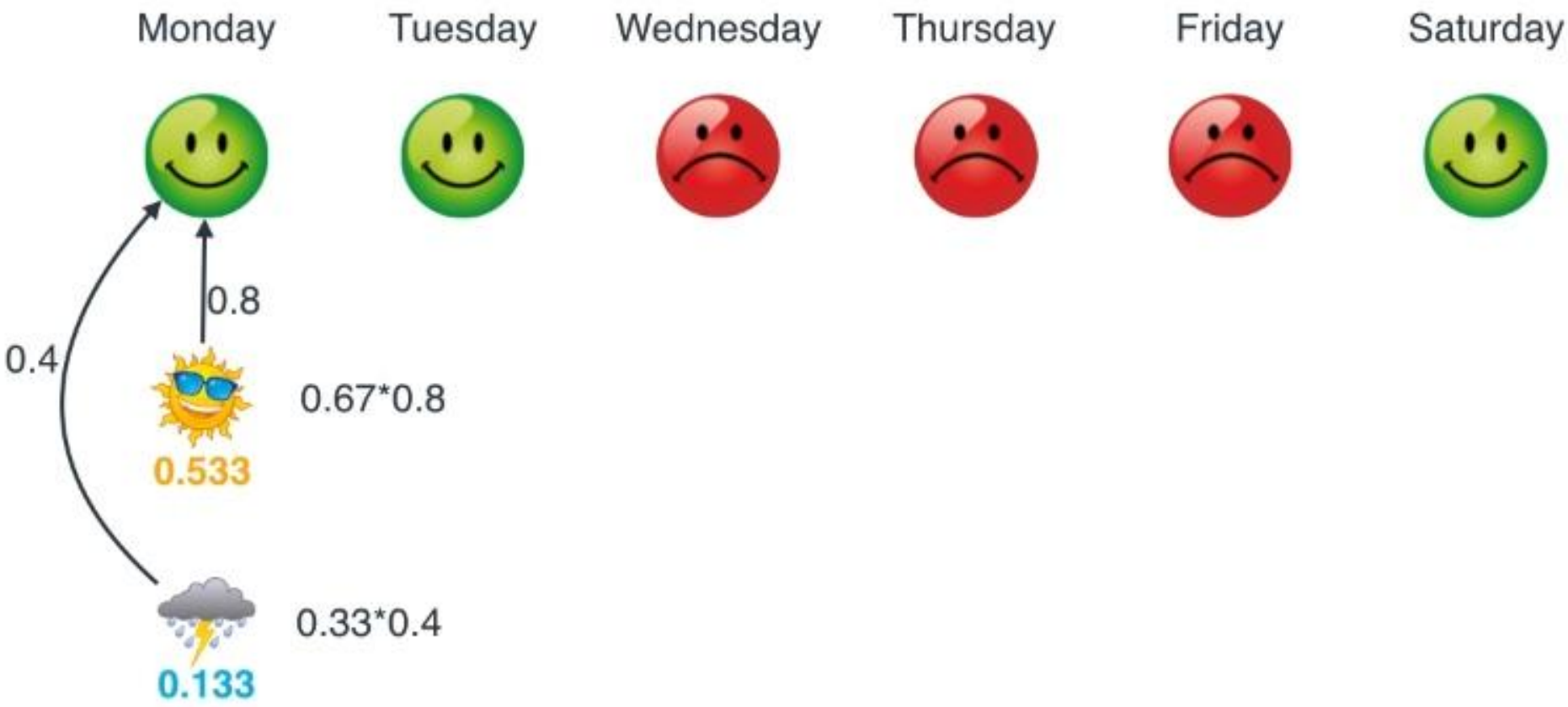
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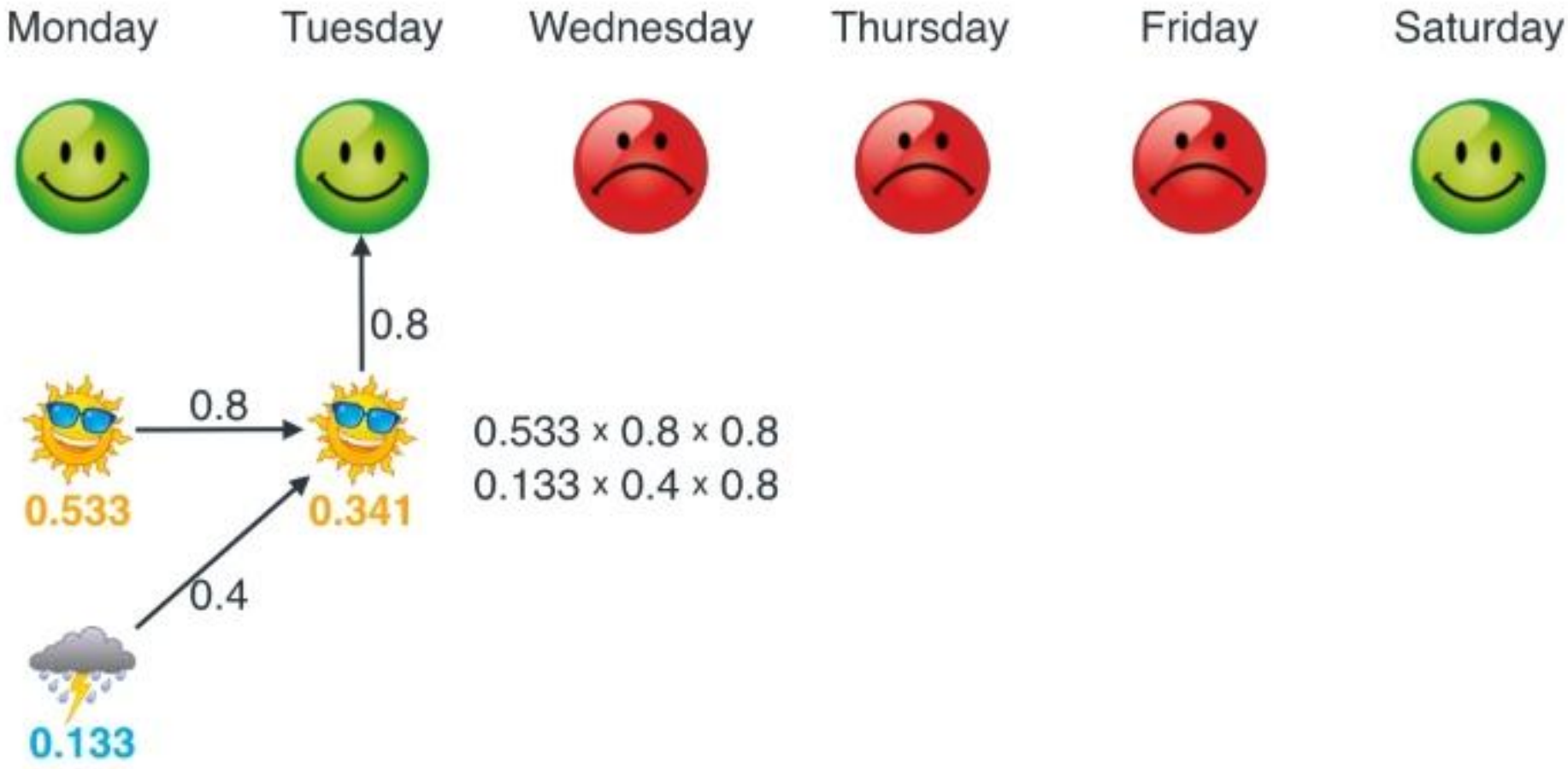
Algoritmul Viterbi



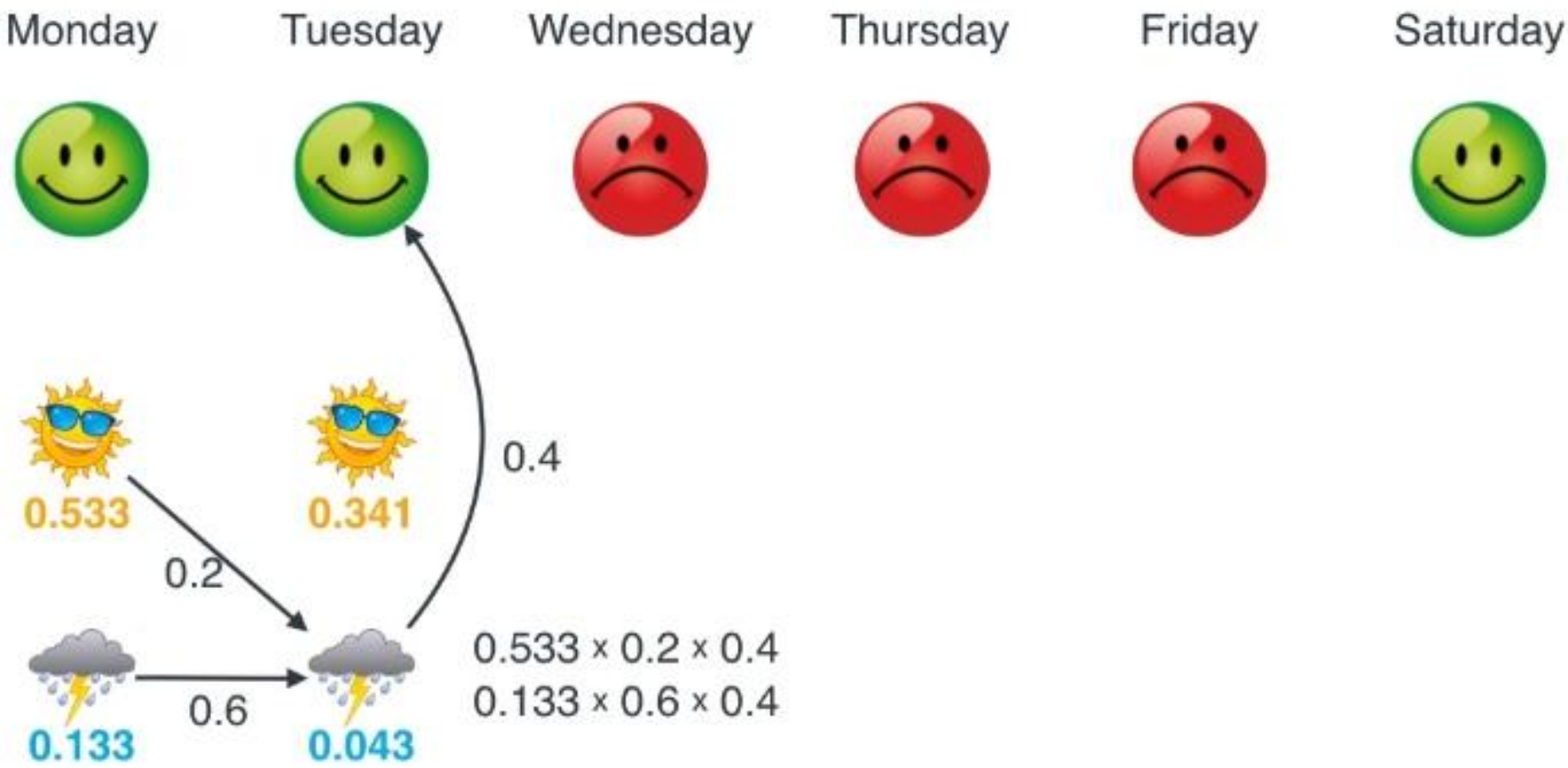
Algoritmul Viterbi



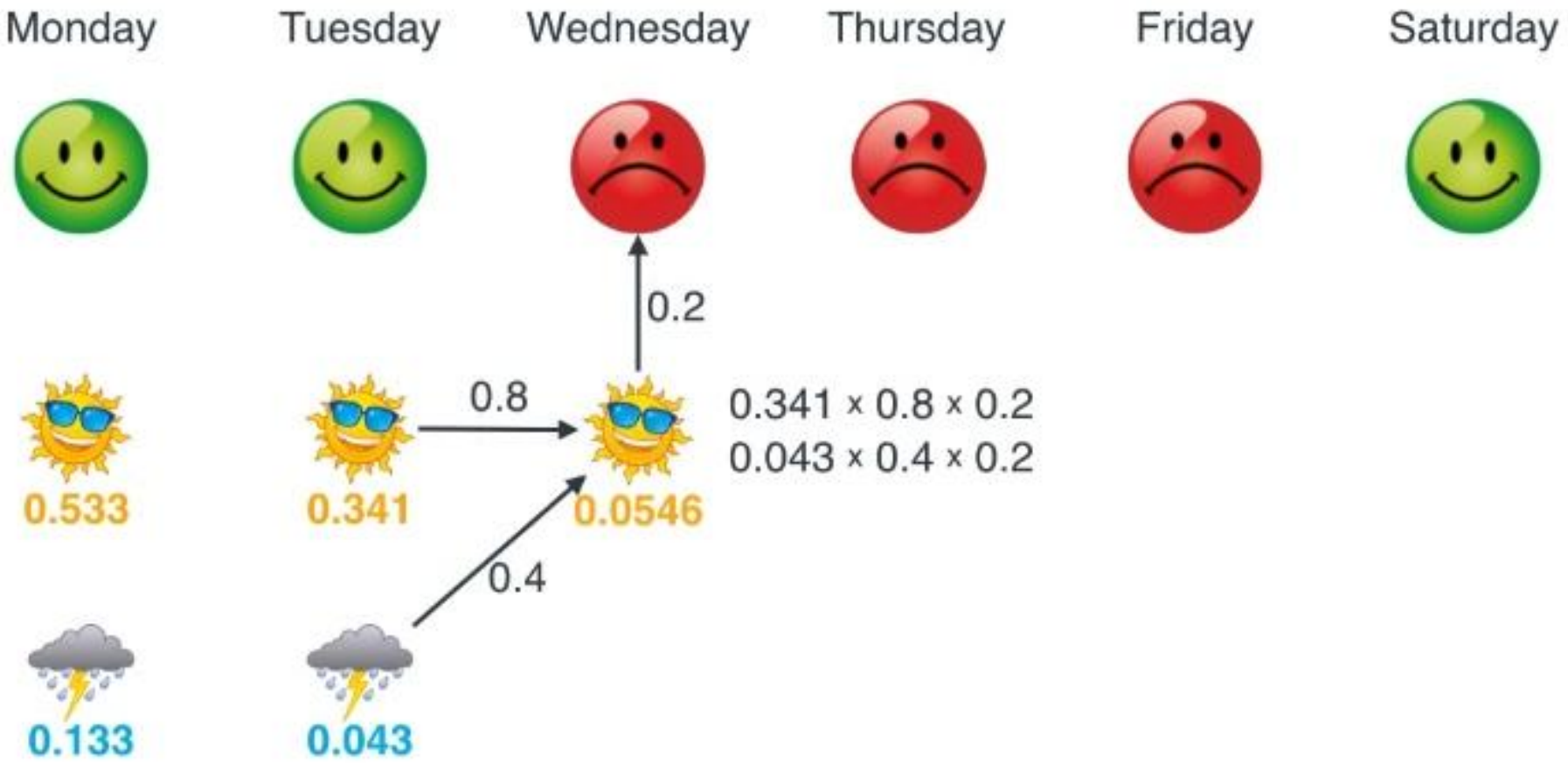
Algoritmul Viterbi



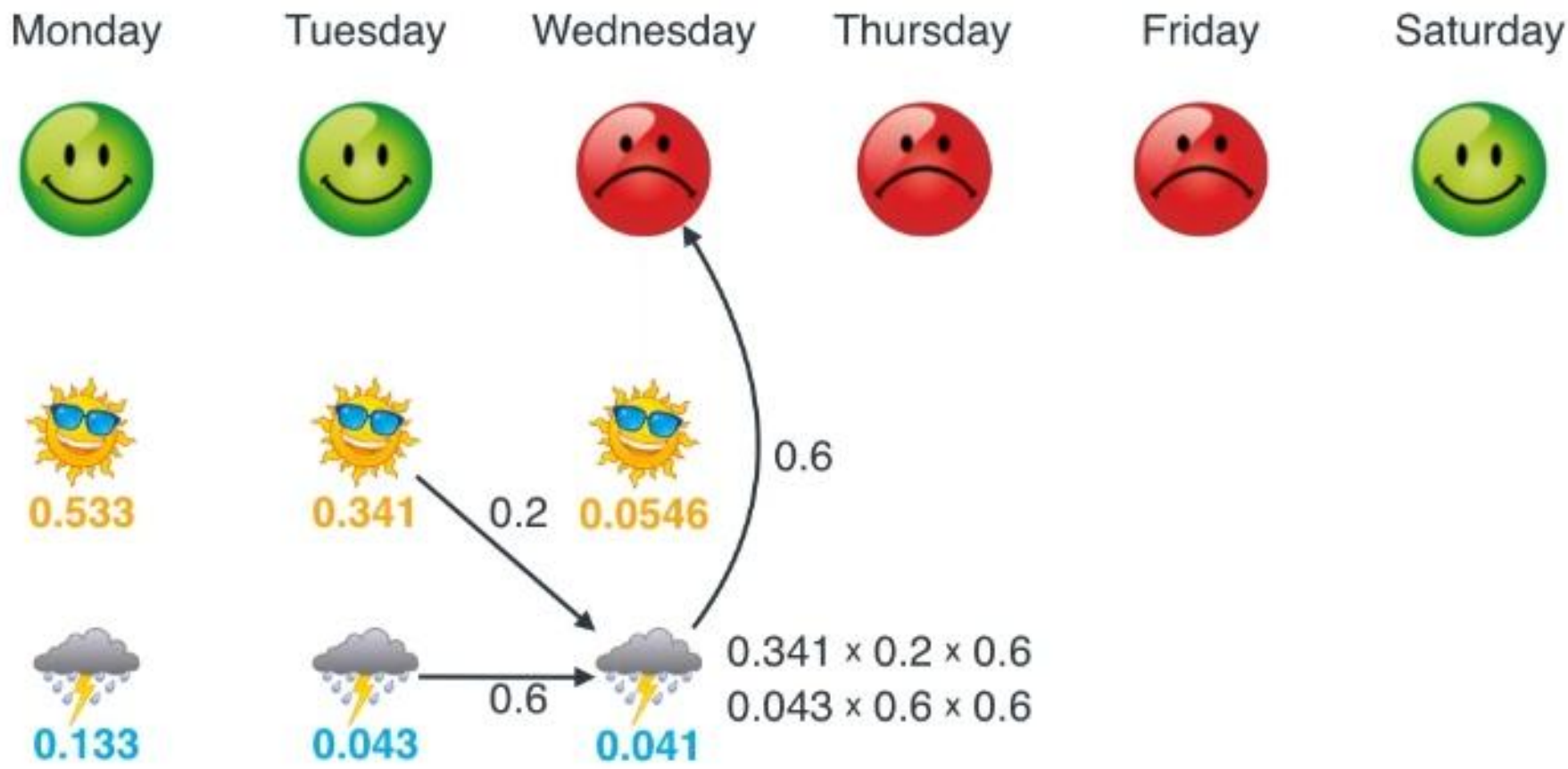
Algoritmul Viterbi



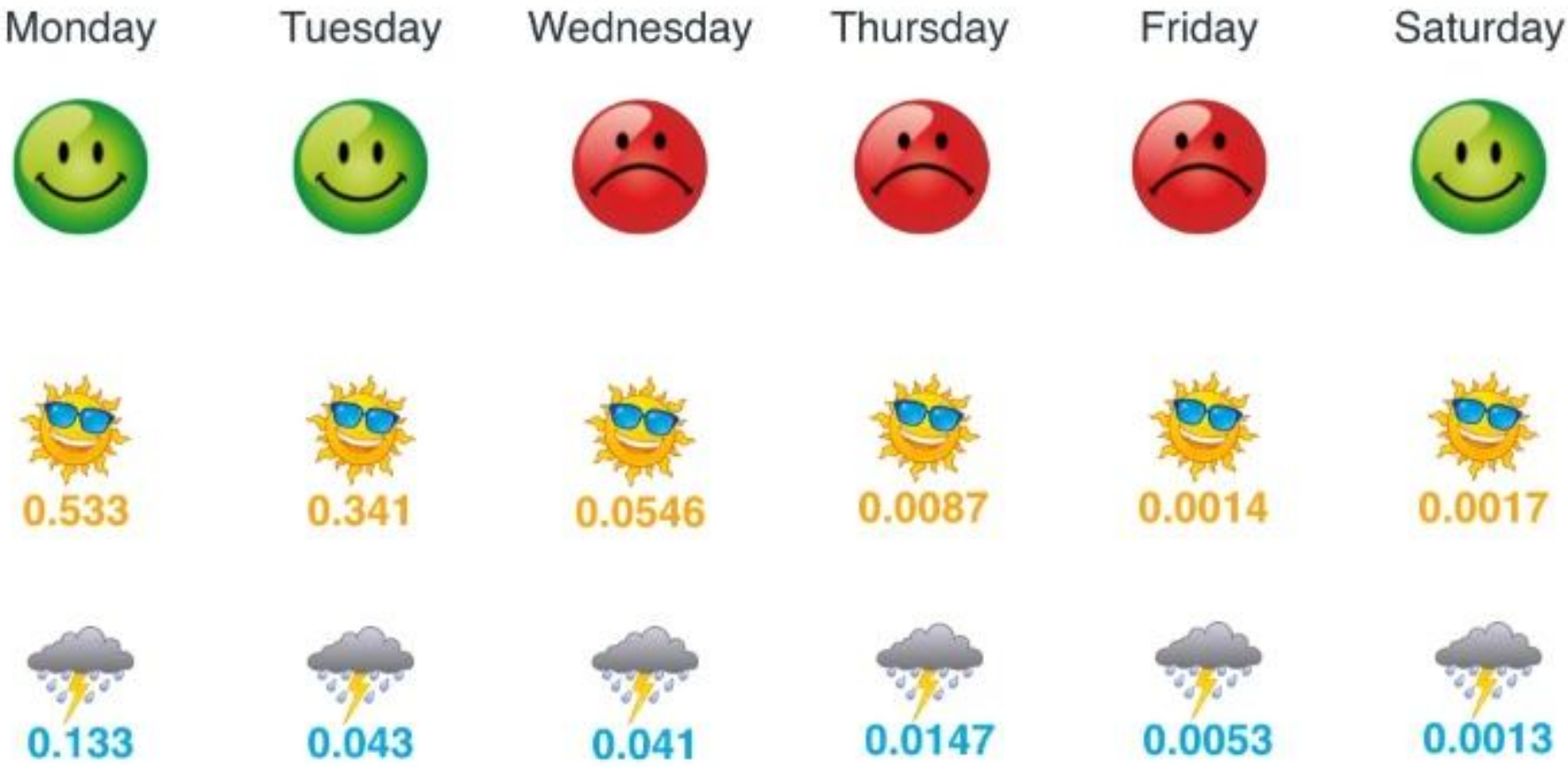
Algoritmul Viterbi



Algoritmul Viterbi



Algoritmul Viterbi



Algoritmul Viterbi

